Investigating the Optimum Tilt Angle for Solar Receiver in Izmir

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Abstract

Nowadays, Renewable Energy Resources (RESs) are used widespread in the most industrial countries like Turkey. Solar energy is deem as one of the most potential RESs; that has gained high attention recently. The performance of any solar collector or photovoltaic model is not just limited to its manufacturing characteristics and used advance technology but it also refers to the other parameters that ensures the maximum solar radiation can be obtained by the solar receiver. In this paper, optimum slope angle for stationary receiver is investigated. Empirical equations of Liu and Jordan method are developed to estimate the optimum angle and maximum solar radiation by using MATLAB software. The case study area and measured data are conducted in Izmir city. The results are obtained per month, season and a year and then are analyzed and compared between monthly average daily and representative day of each month. The comparison is yielded that, the difference of maximum radiation on tilt plane does not exceeds 2.6% due to correlation coefficient R^2.

Key words: Tilt angle, Maximum solar radiation, Liu and Jordan method, correlation coefficient R^2, Izmir.

ABBREVIATIONS

S	Day length	θ_z	Zenith angle
S_o	The maximum possible daily hours of bright sunshine	ω_s	Sunshine hour angle
		ω_{ss}, ω_{sr}	Sunset , sunrise hour angles
θ	Incidence angle	β	Inclination angle
∅	Latitude angle	γ	Surface azimuth angle
ω	Hour angle	δ	Solar declination angle

1. Introduction

The Renewable Energy Sources (RESs) are considered as promising future for the clean global environment. Of all the renewable sources of available energy, solar energy is seen as the most potential source that can be well exploited and invested. It is because of the terrestrial solar radiation is always extremely available and exceeds a human needs. However this depends on the knowledge of distributive intensity of solar radiation on desired sites. There two methods are used to take advantage of solar energy either in PV or thermal systems. In spite manufacturing characteristics and developed advanced technology ply a major roles in high solar performance, extraction of solar energy is also restricted to the optimum tilt and surface azimuth angles for solar receiver. The employed receiver is preferred to be stationary because of low cost, maintenance and un-required energy in contrast with single and dual axial tracking systems [1].

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Many studied and researches were conducted to develop empirical models for optimum inclination and orientation angles in order to foster the calculation of incident solar radiation on an absorbed surface. Generally, mathematical models have similar procedure but are different in term of sky-diffuse solar radiation on tilt surface [2]. It is worth noting that, even though tilt angle may be same to all solar planes, these models do not often give similar predicted solar radiation. In addition, the computed high radiation does not always guarantee a good model but sometimes low solar radiation can be preferable in use because of it is realistic and may have small statistic error like Badescu model [3]. For tilt surface, Reindl model is also used where both slope and orientation angles were investigated as effected parameters in building-integrated photovoltaic (BIPV) performance [4]. For fixed flat plate collector, orientation angle is commonly used 0 or 180 due to southern hemisphere and northern hemisphere respectively [1]. Some authors propose two optimal tilt angles like in summer (latitude −15°) and in winter (latitude −15°) [5].

In this paper, Liu and Jordan (LJ) method is applied to investigate the maximum solar radiation collected by fixed tilt absorber. It is based on isotropic model where is employed because of understandable, reliability and simplicity in use. The essential objective of this study is to investigate each of optimum slope angle per month, season and year. The surface azimuth angle is fixed 0° for facing directly the equator.

This paper is organized up to seven sections: 2. Solar radiation calculations; 3. Liu and Jordan (LJ) method; 4. correlation coefficient R^2; 5. Measured and Calculated Solar Parameters; 6. Results and Discussions; 7. Conclusion.

2. Solar Radiation Calculations

Measurable incident solar radiation on tilt surface is rarely available. Because of this, global solar radiation are initially measured on horizontal plane which may be collected from meteorological stations and then collective data of horizontal is transposed to estimate absorbed radiation on tilt surface. There are widely two type of representative solar radiation. the monthly average daily global solar radiation on a horizontal surface (H) and the hourly total radiation on a horizontal surface (I) for each hour for extended periods such as one or more years.

2.1. Solar Radiation on Horizontal Plane

Monthly mean daily extraterrestrial global radiation on a horizontal surface (H_o) with unit MJ/m^2-day can be calculated as follows [6][7]:

$$H_o = \frac{24*3600}{\pi} G_{on} \left(cos\, \emptyset \, cos\, \delta \, sin\, \omega_s + \frac{\pi}{180} \omega_s \, sin\, \emptyset \, sin\, \delta \right) \quad \text{(a.1)}$$

$$G_{on} = G_{sc} \left(1 + 0{,}033\, cos\left(\frac{360*n}{365} \right) \right) \quad \text{(b.1)}$$

The G_{on} extraterrestrial radiation measured on the plane normal to the radiation on the n day of a year W/m^2. Sometimes n indicates the representative day of each month like January and February are17 and 16 respectively, for $|\emptyset| > 66.5^o$ whish is valid in our case study [8]. G_{sc}'s value of 1376 W/m^2 is adopted by World Radiation Center (WRC) with uncertainty order of 1%

[9]. The declination angle is calculated by:

$$\delta = 23{,}45\, sin\left(\frac{360}{365}(284 + n)\right) \tag{2}$$

$$\omega_s = cos^{-1}(-tan\, \emptyset\, tan\, \delta) \tag{3}$$

Usually, incident global radiation (H) on horizontal surface is presented in two parameters; beam (H_b) and diffuse (H_d) radiations with unit MJ/m^2-day as follows:

$$H = H_b + H_d \tag{4}$$

In the presence of atmosphere, the solar radiation is exposed into scattering and absorption, therefore monthly average clearness index is introduced K_T:

$$K_T = \frac{H}{H_o} \tag{5}$$

The variability of solar models due to estimation of tilt angle is related to disagreement within those empirical relationships such sky-diffuse correlation should be used. H_d/H , H_d/H and H/H_o equations are developed by Ulgen and Hepbasli are used as shown below [10][11]:

$$\frac{H_d}{H_o} = 0.1155 + 0.1958(K_T) \tag{6}$$

$$\frac{H}{H_o} = 0.2408 + 0.3625\left(\frac{S}{S_0}\right) + 0.4597\left(\frac{S}{S_0}\right)^2 - 0.3708\left(\frac{S}{S_0}\right)^3 \tag{7}$$

$$\frac{H_d}{H} = 0.6595 - 0.7841(\frac{S}{S_0}) + 0.2579(\frac{S}{S_0})^2 \tag{8}$$

$$S_o = \frac{2}{15}\omega_s \tag{9}$$

2.2. Solar Radiation on Slope Plane

The amount of solar radiation received by slope plane is sum of direct beam ($H_{T,b}$), diffuse ($H_{T,d}$) and ground reflected ($H_{T,r}$) radiations as described below:

$$H_T = H_{T,b} + H_{T,d} + H_{T,r} \tag{10}$$

2.2.1. Beam Radiation

The first component of inclined plane is $H_{T,b}$ which is quite easy to be computed compare with other two parts. Equations of (11) and (12) are visible formulas for all direct beam radiation.

$$H_{T,b} = H_b R_b \tag{11}$$

$$H_b = H - H_d \tag{12}$$

$$R_b = \frac{\cos\theta}{\cos\theta_z} = \left[\frac{\begin{matrix}\sin\delta\sin\emptyset\cos\beta - \sin\delta\cos\emptyset\sin\beta\cos\gamma \\ \cos\delta\cos\emptyset\cos\beta\cos\omega + \cos\delta\sin\emptyset\sin\beta\cos\gamma\cos\omega \\ +\cos\delta\sin\beta\sin\gamma\sin\omega\end{matrix}}{\cos\emptyset\cos\delta\sin\omega + \sin\emptyset\sin\delta}\right] \tag{13}$$

Generally, beam radiation tilt factor *(r_b)* is defined as the ratio of monthly average daily beam radiation on inclined surface to horizontal surface at any time. It is function of transmittance of the atmosphere, contains water vapor , cloudiness and particulate concentration.

2.2.2. Diffuse radiation

This second component of solar radiation is considered as serious part in collective solar radiation due to difficulty of anticipated its directions. Therefore $H_{T,d}$ consists of isotropic ($H_{d,iso}$), circumsolar ($H_{d,cs}$) and horizon brightening ($H_{d,hz}$) associated with radiation view factor (F) from the sky to the tilted surface as shown below [1][3]:

$$H_{T,d} = H_{d,iso}\, F_{c-s} + H_{d,cs}\, R_b + H_{d,hz}\, F_{c-hz} \tag{14}$$

There several models are used to anticipate the diffuse radiation on inclined surface, where are categorized into isotropic and anisotropic sky models. In our case of sky diffuse, isotropic model is implemented as shown below:

$$H_{T,d} = H_{d,iso}\, \frac{1+\cos\beta}{2} \tag{15}$$

2.2.3. Reflected radiation

In case of isotropic reflection, the reflected global radiation by ground when is perfectly diffuse reflector like concrete floor; can be expressed as:

$$H_{T,r} = H_b\, \rho_b + H_d\, \rho_d \left(\frac{1-\cos\beta}{2}\right) \tag{16}$$

When ρ_b(for direct) and ρ_d(for sky-diffuse) are identical, ρ is used as common variable and latter equation is reduced into [1]:

$$H_{T,r} = H\, \rho \left(\frac{1-\cos\beta}{2}\right) \tag{17}$$

ρ values depend on reflective surface. In case of grounded snow, surface albedo is ranging between 0.35 for old snow into 0.95 for dry new snow. For absence of snow, surface albedo is usually 0.2 [4].

3. Liu and Jordan (LJ) model

In 1960, Liu and Jordan proposed method to estimate the three components of solar radiation collected by slope plane. The sky-diffuse radiation was assumed to be isotropic while circumsolar and horizon brightening are zero. Mathematically it is presented as below [9][8][12]:

$$H_T = H_b R_b + H_d \left(\frac{1+\cos\beta}{2}\right) + H\rho_g \left(\frac{1-\cos\beta}{2}\right) \quad (18)$$

For simplicity, R_b was redefined by Liu and Jordan as ratio daily extraterrestrial solar radiation on inclined plane due to horizontal plane for surface azimuth angle γ = 0º. [2][9].

$$R_b = \left[\frac{\cos(\emptyset-\beta)\cos\delta\sin\omega'_s + \frac{\pi\omega'_s}{180}\sin(\emptyset-\beta)\sin\delta}{\cos\emptyset\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}\sin\emptyset\sin\delta}\right] \quad (a.19)$$

$$\omega'_s = min\begin{bmatrix}\omega_s \\ cos^{-1}(-tan\,(\emptyset-\beta)\,tan\,\delta)\end{bmatrix} \quad (b.19)$$

4. Correlation Coefficient R²

In order to estimate the difference between different results, correlation coefficient R^2 is used to determine the linear relationship between the measured and estimated data by using formula as below [11]:

$$R^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})}{\sqrt{[\sum_{i=1}^{n}(y_i-\bar{y})^2][\sum_{i=1}^{n}(x_i-\bar{x})^2]}} \quad (20)$$

$\bar{x}$ and $\bar{y}$ denote the mean measured and estimated values respectively. *n* represents the number of event in this work is 12 related to the number of month.

5. Measured and Calculated Solar Parameters

In this work, Izmir is the third biggest city of population in Turkey; therefore, it is taken as the area of case study with latitude of 38.46º where the solar and wind meteorological station is located in. The measuring instruments are positioned on the roof of solar energy institute of Ege university. The collective measured data of solar radiation is from January of 2013 up to December of 2015. In the Table 1, *H* is measured while H_o, K_T and H_d are calculated by equations (1), (5) and (6) respectively. In this Table 1, n presents the whole days of a year (365 day) and then average for every month is computed.

In Table 2, *S* is measured while H_o, H, H_d and S_o are computed regarding into equations of (1) (7) (8) and (9) respectivily. In this Table 2, n is just peresented by one recommonded day of each month for example; January =17 and Desember = 344 day of a year [13].

Table 1: Illustration of monthly mean daily for n including all days of a year

Month	H_o MJ/m² day	H MJ/m² day	H_d MJ/m² day	K_T
Jan	16.2	7.38	3.33	0.46
Feb	21.16	10.27	4.47	0.49
Mar	28.12	14.16	6	0.5
Apr	35.04	19.29	7.82	0.55
May	39.79	23.55*	9.19	0.59
Jun	41.69	25.66	9.88	0.62
Jul	40.65	27.55	10.11	0.68
Aug	36.71	24.11	8.98	0.66
Sep	30.42	19.07	7.27	0.63
Oct	23.18	14.06	5.45	0.61
Nov	17.28	9.44	3.86	0.55
Dec	14.69	7.51	3.16	0.51

*May' data of 2015 was not exist, therefore average May' data of 2013 and 2014 were taken.

Table 2: It represents solar parameters for n is denoted by one recommanded day of each month

Month	H_o MJ/m² day	S_o	S**	H MJ/m² day	H_d MJ/m² day
Jan	16.16	9.64	4.39	7.53	2.68
Feb	21.41	10.6	5.35	10.56	3.48
Mar	28.13	11.74	6.30	14.36	4.49
Apr	35.03	13.01	7.67	18.86	5.41
May	39.81	14.09	9.29	22.83	5.81
Jun	41.72	14.64	11.55	26.32	5.3
Jul	40.7	14.39	12.25	26.61	4.76
Aug	36.81	13.46	11.35	23.96	4.35
Sep	30.58	12.23	9.45	19.09	3.96
Oct	23.35	10.97	7.18	13.33	3.42
Nov	17.37	9.89	5.18	8.75	2.8
Dec	14.71	9.37	3.38	6.09	2.5

6. Results and Discussions

The estimation of proper tilt angle and maximum solar energy per month, season and a year are presented and two cases are taken in consideration.

First Case with Table 1: it considers the entire days of year where n is ranging between 1 and 365 day. In Table 3, the optimum tilt mean angle β^o_{opt} and mean maximum radiation on slope plane $H_{T,opt}$ are presented for each month. It is cleared that, the angle varies from 0° up to 63.77° . $H_{T,opt}$ is minimum at January and maximum at July. It is worth mentioning that, the β^o_{opt} is incremented by 1° up to 90° per day through the calculations.

In Table 4, season and year average of β^o_{opt} and $H_{T,opt}$ are showed. β^o_{opt} is caclulated by taking average of 3 months corrossponding to each season and that yeiled to have four seasonal angle in; winter = 58.68^o, spring = 20.86^o, summer = 5.58^o, fall = 49^o. Annual tilt angle also is produced 32.91^o.

Table 3: Illustration of monthly mean daily of β^o_{opt} and $H_{T,opt}$

Month	β^o_{opt}	$H_{T,opt}$	Month	β^o_{opt}	$H_{T,opt}$
Jan	60.48	12.53	Jul	1.9	27.57
Feb	51.79	14.65	Aug	14.84	24.72
Mar	37	16.73	Sep	31.7	21.55
Apr	20.03	20.19	Oct	48.45	19.25
May	5.55	23.65	Nov	59.37	15.85
Jun	0	25.66	Dec	63.77	14.04

Table 4: Showing β^o_{opt} and $H_{T,opt}$ for seaon and year

Month	Season	β^o_{opt}	$H_{T,opt}$		β^o_{opt}	$H_{T,opt}$
Dec	Winter	58.68	14.00	Annual	32.91	12.39
Jan			12.52			11.37
Feb			14.55			14.01
Mar	Spring	20.86	16.18			16.65
Apr			20.15			19.81
May			23.11			21.98
Jun	Summer	5.58	25.53			22.95
Jul			27.51			25.11
Aug			24.45			23.86
Sep	Fall	46.51	20.99			21.49
Oct			19.2			18.63
Nov			15.5			14.46

Second Case with Table 2: It is similar to case one. It also presents β^o_{opt} and $H_{T,opt}$ but n represents the recomanded day of each month. Table 5 shows the maximum radiation corrossponding with optimum slope angle. Meanwhile, Table 6 denotes the season and annual of both β^o_{opt} and $H_{T,opt}$.

Table 5: Denoting β^o_{opt} and $H_{T,opt}$ for representative day

Month	β^o_{opt}	$H_{T,opt}$	Month	β^o_{opt}	$H_{T,opt}$
Jan	63	13.96	Jul	1	26.61
Feb	54	15.99	Aug	17	24.76
Mar	40	17.61	Sep	35	22.4
Apr	22	19.96	Oct	51	19.46
May	6	22.92	Nov	61	15.72
Jun	0	26.32	Dec	64	11.49

Table 6: Optimum tilt angles and maximum solar radiation for season and year.

Month	Season	β^o_{opt}	$H_{T,opt}$		β^o_{opt}	$H_{T,opt}$
Dec	Winter	60.33	11.47	Annual	34.5	10.25
Jan			13.95			12.55
Feb			15.91			15.24
Mar	Spring	22.67	16.98			17.55
Apr			19.96			19.62
May			22.3			21.09
Jun	Summer	6	26.14			22.93
Jul			26.56			23.76
Aug			24.42			23.86
Sep	Fall	49	21.84			22.4
Oct			19.44			18.77
Nov			15.42			14.29

In Table 7 and Figure 1, corrolation coeffiecient R^2 is used to identify the accuracy between phase one and phase two. It is worth mentionig that, the difference between them are tiny as in monthly average daily $H_{T,opt}$ = 0.974% and R^2 = 2.6% which allwos phase two presents phase one in further simple and fact calculations.

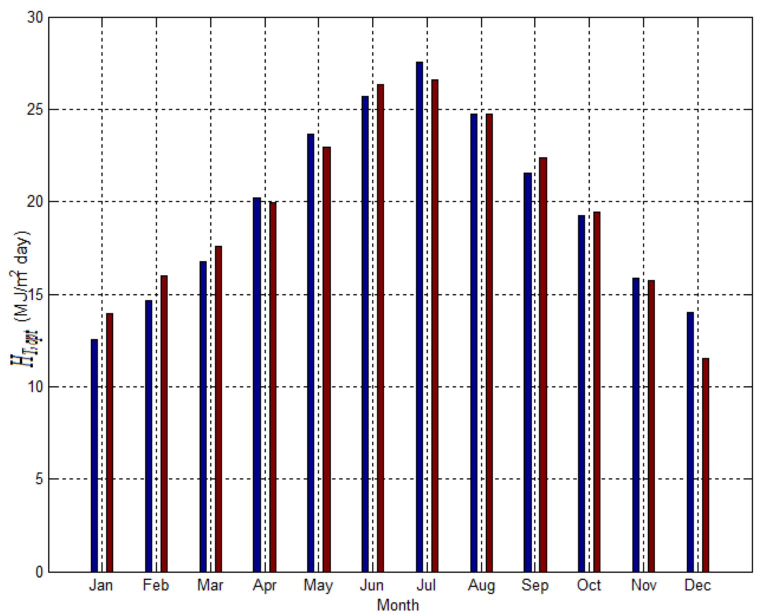

Figure 1. It represents monthly mean daily $H_{T,opt}$ (blue) and $H_{T,opt}$ for representative day (red)

Table 7: Corrolation coeffiecient R^2

H_o	H	H_d	$H_{T,opt}$	β^o_{opt}	$H_{T,opt\ annual}$
0.999	0.996	0.887	0.974	0.998	0.975

7. Conclusion

Determination of optimum tilt angle and maximum solar radiation form an importance issue in solar system design. Therefore, Liu and Jordan method are used. The obtained results presents optimum tilt angle and maximum solar per month, season and a year in Izmir city. The conducted calculations are based on; 1) monthly mean daily solar radiation and 2) recommended specific day of each month. The comparison shows that the difference of maximum radiation on tilt plane is small due to $R^2 = 2.6\%$. It is concluded that presenting each month by one recommended day can be used.

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